CHAPTER 2: The Derivative
Concepts/Skills to know:

- **Continuous functions** and **discontinuities** (removable, jump, or infinite).
- **Secant lines** through 2 points related to the graph of ***y = f(x)*** $x_1=a$ and $x_2=a+h$ $y_1=f(a)$ and $y_2=f(a+h)$
  slope = **average** rate of change of ***y*** with respect to ***x*** The value of ***h*** is the distance between ***a*** and ***a+h.***
  $$m = \frac{f(a+h)-f(a)}{(a+h)-a} = \frac{f(a+h)-f(a)}{h}$$
  slope of line through 2 points ***(a, f(a))*** and ***((a+h), f(a+h))***
- **Tangent lines** through 1 point related to the graph of ***y = f(x)***
  slope = **instantaneous** rate of change of **y** with respect to **x** at specific point (***a, f(a))***
  $$m_a = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = f'(a)$$ if it exists. Use algebra to work through this!

  f'(a) is slope of tangent line through ***(a, f(a))***. Equation of tangent line thru (x_1, y_1): $\frac{y-y_1}{x-x_1} = m_a$ (solve for y)

- **Velocity (or speed) and Distance (or position)**

s(t)=0 when distance = 0 and *s'(t)=0* when instantaneous velocity=0

  Instantaneous velocity = instantaneous rate of change of position with respect to time
  If ***s(t)*** is the distance function (distance at time ***t***),
  then ***s'(t)*** is the instantaneous velocity function (instantaneous velocity at time ***t***).
  $$s'(t) = \lim_{h\to 0}\frac{s(t+h)-s(t)}{(t+h)-t} = \lim_{h\to 0}\frac{s(t+h)-s(t)}{h}$$
  Remember: **average velocity** $= \frac{s(t_2)-s(t_1)}{t_2-t_1}$
- **Limit definition of the derivative function**. (Derivative-value depends on the value of ***x***).
  $$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{(x+h)-x} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$ if it exists. Use algebra to work through this!
- ***f'(x) = 0*** where line tangent to graph of ***f(x)*** is horizontal (slope=0) at point of tangency ***(x, f(x))***. *Solve for **x** and calculate **f(x)**.*
- **Shortcuts for Differentiation** ***c, m, b, n*** are constants; ***f, g, h*** are functions of ***x***.

You may use shortcuts <u>unless</u> you're told to use the limit definition of derivative.

h(x) function *with respect to **x***	***h'(x) derivative*** *with respect to **x***	**Comments**
c	0	derivative of constant function
$m\cdot x + b$	m	derivative of linear function
x^n	$n\cdot x^{n-1}$	power rule
$c\cdot x^n$	$(c\cdot n)x^{n-1}$	constant multiple rule
$c\cdot f$	$c\cdot f'$	constant multiple rule
$f + g$	$f' + g'$	sum rule
$f - g$	$f' - g'$	difference rule
$f \cdot g$	$f'\cdot g + f\cdot g'$	product rule
$\frac{f}{g}$	$\frac{g\cdot f' - f\cdot g'}{g^2}$	quotient rule

- **Derivative Notation** $\frac{d}{dx}$ and D_x are **differential operators**.

First derivative: $y' = f'(x) = \frac{dy}{dx} = \frac{d}{dx}(f(x)) = D_x y$

Second derivative: $y'' = f''(x) = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = \frac{d^2}{dx^2}(f(x)) = D^2_x y$

Third derivative: $y''' = f'''(x) = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3} = \frac{d^3}{dx^3}(f(x)) = D^3_x y$

- **Differentiation** is the process of finding a derivative. **difference quotient** $= \frac{difference\_of\_outputs}{difference\_of\_inputs}$